The TREND IN ENGINEERING

at the University of Washington

In this issue:

PLANE PLASTICITY PROBLEMS AND THE DIRECT STIFFNESS METHOD

A RATIONAL ANALYTIC THEORY OF FATIGUE

HIGH-TEMPERATURE METALLOGRAPHY

PUBLICATIONS AND THESES COLLEGE OF ENGINEERING 1959-1960

January, 1961

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1) It stimulates, promotes, and coordinates investigations and research in all fields of engineering.
2) It publishes results of significant research achievements.
3) It provides opportunities through Graduate Research Assistantships for engineering students to extend their professional education while pursuing a course of study leading to the M.S. or Ph.D. degree.

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at the University of Washington

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CONTENTS

In Retrospect: How the First Atomic Bomb Affected One of Its Creators Fifteen Years Ago (From the H. D. Smyth Report) . . . 1

Research Assistantships for 1961-1962 2

Plane Elasticity Problems and the Direct Stiffness Method *Harold C. Martin* 5

A Rational Analytic Theory of Fatigue *Paul C. Paris, Mario P. Gomez, William E. Anderson* . . 9

High-Temperature Metallography *Maurice B. Kasen, Douglas H. Polonis* 15

Publications and Graduate Theses, College of Engineering: 1959-1960 20

New NSF Basic-Research Grant 24

IN RETROSPECT: HOW THE FIRST ATOMIC BOMB AFFECTED ONE OF ITS CREATORS FIFTEEN YEARS AGO*

PRESENT OVERALL STATUS

As the result of the labors of the Manhattan District organization in Washington and in Tennessee, of the scientific groups at Berkeley, Chicago, Columbia, Los Alamos, and elsewhere, of the industrial groups at Clinton, Hanford, and many other places, the end of June 1945 finds us expecting from day to day to hear of the explosion of the first atomic bomb devised by man. All the problems are believed to have been solved at least well enough to make a bomb practicable. A sustained neutron chain reaction resulting from nuclear fission has been demonstrated; the conditions necessary to cause such a reaction to occur explosively have been established and can be achieved; production plants of several different types are in operation, building up a stock pile of the explosive material. Although we do not know when the first explosion will occur nor how effective it will be, announcement of its occurrence will precede the publication of this report. Even if the first attempt is relatively ineffective, there is little doubt that later efforts will be highly effective; the devastation from a single bomb is expected to be comparable to that of a major air raid by usual methods.

A weapon has been developed that is potentially destructive beyond the wildest nightmares of the imagination; a weapon so ideally suited to sudden unannounced attack that a country's major cities might be destroyed overnight by an ostensibly friendly power. This weapon has been created not by the devilish inspiration of some warped genius but by the arduous labor of thousands of normal men and women working for the safety of their country. Many of the principles that have been used were well known to the international scientific world in 1940. To develop the necessary industrial processes from these principles has been costly in time, effort, and money, but the processes which we selected for serious effort have worked and several that we have not chosen could probably be made to work. We have an initial advantage in time because, so far as we know, other countries have not been able to carry out parallel developments during the war period. We also have a general advantage in scientific and particularly in industrial strength, but such an advantage can easily be thrown away.

. . . . . .

PROGNOSTICATION

As to the future, one may guess that technical developments will take place along two lines. From the military point of view it is reasonably certain that there will be improvements both in the processes of producing fissionable material and in its use. It is conceivable that totally different methods may be discovered for converting matter into energy since it is to be remembered that the energy released in uranium fission corresponds to the utilization of only about one-tenth of one per cent of its mass. Should a scheme be devised for converting to energy even as much as a few per cent of the matter of some common material, civilization would have the means to commit suicide at will.

The possible uses of nuclear energy are not all destructive, and the second direction in which technical development can be expected is along the paths of peace. In the fall of 1944 General Groves appointed a committee to look into these possibilities as well as those of military significance. This [Tolman] committee . . . received a multitude of suggestions . . . While there was general agreement that a great industry might eventually arise, comparable, perhaps, with the electronics industry, there was disagreement as to how rapidly such an industry would grow; the consensus was that the growth would be slow over a period of many years. At least there is no immediate prospect of running cars with nuclear power or lighting houses with radioactive lamps although there is a good probability that nuclear power for special purposes could be developed within ten years and that plentiful supplies of radioactive materials can have a profound effect on scienitfic research and perhaps on the treatment of certain diseases in a similar period.

PLANNING FOR THE FUTURE

During the war the effort has been to achieve the maximum military results. It has been apparent for some time that some sort of government control and support in the field of nuclear energy must continue after the war. Many of the men associated with the project have recognized this fact and have come forward with various proposals, some of which were considered by the Tolman Committee, although it was only a temporary advisory committee reporting to General Groves. An interim committee at a high level is now engaged in formulating plans for a continuing organization. This committee is also discussing matters of general policy about which many of the more thoughtful men on the project have been deeply concerned since the work was begun and especially since success became more and more probable.

THE QUESTIONS BEFORE THE PEOPLE

We find ourselves with an explosive which is far from completely perfected. Yet the future possibilities of such explosives are appalling, and their effects on future wars and international affairs are of fundamental importance. Here is a new tool for mankind, a tool of unimaginable destructive power. Its development raises many questions that must be answered in the near future.

Because of the restrictions of military security there has been no chance for the Congress or the people to debate such questions. They have been seriously considered by all concerned and vigorously debated among the scientists, and the conclusions reached have been passed along to the highest authorities. These questions are not technical questions; they are political and social questions, and the answers given to them may affect all mankind for generations. In thinking about them the men on the project have been thinking as citizens of the United States vitally interested in the welfare of the human race. It has been their duty and that of the responsible high government officials who were informed to look beyond the limits of the present war and its weapons to the ultimate implications of these discoveries. This was a heavy responsibility. In a free country like ours, such questions should be debated by the people and decisions must be made by the people through their representatives. This is one reason for the release of this report. It is a semitechnical report which it is hoped men of science in this country can use to help their fellow citizens in reaching wise decisions. The people of the country must be informed if they are to discharge their responsibilities wisely.

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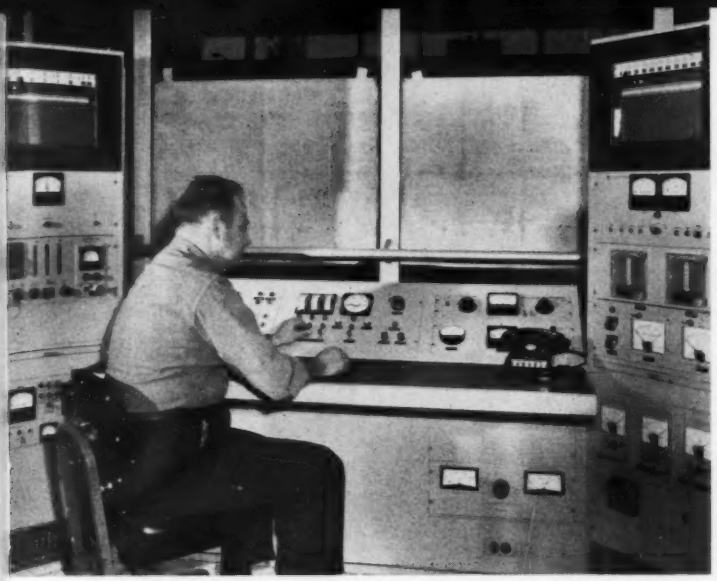
* The content of this page is from the conclusion of the first public report (printed in 1945) on the development of the atomic bomb. Except for short omissions, this excerpt includes all the "General Summary" of H. D. Smyth's report, *A General Account of the Development of Methods of Using Atomic Energy for Military Purposes under the Auspices of the United States Government*, U.S. Government Printing Office, 1945.



At the control and instrument console for the new UW Training Reactor

THE ENGINEERING EXPERIMENT STATION announces the availability of research assistantships for 1961-62. Appointees must qualify for full graduate standing in the University. A graduate assistant receives $2,700.00 over four quarters at the rate of $225.00 per month; he pays tuition, but receives a waiver for the difference between resident and non-resident fees. These research assistantships require that half time be devoted to research leading to a thesis.

Research opportunities are offered in all branches of engineering, including nuclear engineering. (See the University of Washington catalog.) Typical examples of current research will be found in this and other issues of *The Trend in Engineering*. Assignments are open in connection with government, industry, and Engineering Experiment Station research projects. Teaching assistantships are also open in some departments.

Additional information and application forms may be obtained from the Director of the Engineering Experiment Station, University of Washington, Seattle 5, Washington. Applications may be submitted at any time, although most appointments become effective September 16, 1961.

Research Assistantships for 1961-1962

South part of the University of Washington campus, where 2810 engineering students are enrolled

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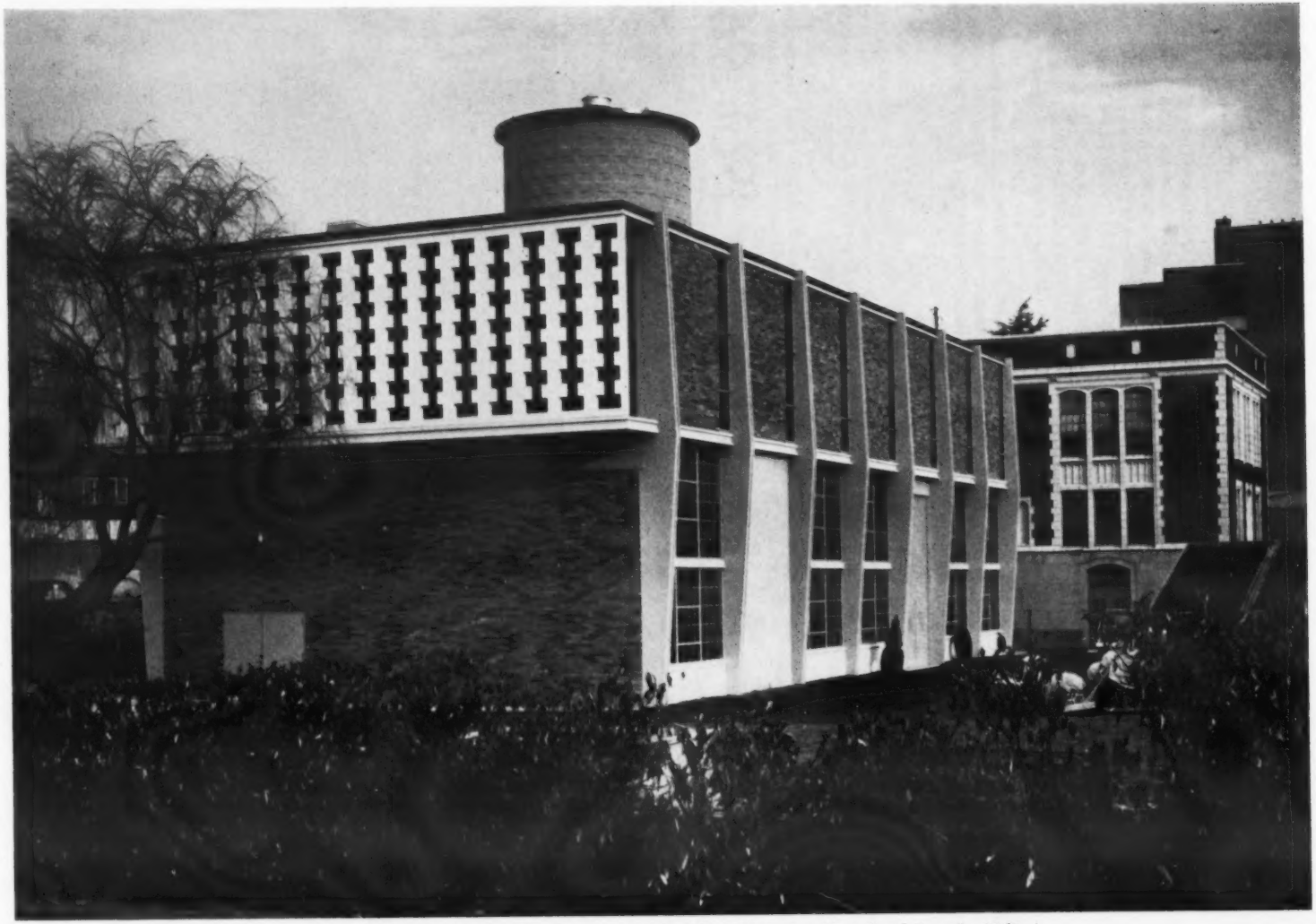
ENGINEERING



The Addition to the Charles W. Harris Hydraulics Laboratory Completed Late in 1960

Plane Elasticity Problems and the Direct Stiffness Method

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H. C. Martin

Plane elasticity problems involve the determination of stresses and deformations in a two-dimensional continuum. Loads and deformations are restricted to those lying within the plane of the continuum.

Solutions are usually achieved by solving the appropriate differential equations, subject to the boundary conditions of the problem. In certain simple cases, exact solutions may be found. More often, approximate solutions are the only ones that can be determined. Examples may be found in any text on elasticity.

The purpose of this article is to call attention to a different concept for finding approximate solutions to such problems. This alternate procedure replaces the continuum with an assemblage of finite elements and then employs the concepts of the direct stiffness method in obtaining a solution. It thereby becomes feasible to analyze very complex problems, and therein lies the real advantage of this method. Digital computing equipment must be used in carrying out the numerical calculations.

The ideas underlying the method were developed as part of a general procedure for analyzing complex engineering structures. (See Reference 1, in which the theory used in this present article was first given.) Details and points of view have been modified to some extent in the present writing.

Direct Stiffness Method

The differential equations of the continuum are useful in reaching a solution only if the problem is quite simple. Engineering structures built up of bars, beams, sheet elements, and so on are generally much too complex to be analyzed in this manner. It then becomes necessary to apply the fundamental conditions of equilibrium and continuity in such a manner that the mathematical formulation is given in terms of algebraic equations. The direct stiffness method offers a convenient scheme for obtaining these equations.

In applying the stiffness method, we assume that the built-up structure can be analyzed as an assemblage of structural units connected at certain selected points called nodes. The simplest example is the pin-jointed truss. More general structures will include beams, plate elements, and so forth as parts of the assemblage. However, even in these cases, the various subelements will be assumed to be connected only at the node points. Later the two-dimensional continuum will also be treated in this same manner.

Before the stiffness method can be applied to the overall structure, the stiffness of the structural subunits must be known. This can be illustrated by reference to the typical truss member. If in Fig. 1,a L = length of member, A = cross-sectional area, E = modulus of elasticity, and X and u are force and displacement as shown, elementary theory then proves that

$$X_2 = -X_1 = \frac{AE}{L}(u_2 - u_1). \tag{1}$$

In Eq. (1) equilibrium is satisfied.

For a general orientation of the member as shown in Fig. 1,b, appropriate coordinate transformation applied to Eq. (1) leads to

$$\begin{Bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} \lambda^2 & & \textit{Symmetric} & \\ \lambda\mu & \mu^2 & & \\ -\lambda^2 & -\lambda\mu & & \\ -\lambda\mu & -\mu^2 & \lambda\mu & \mu^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \tag{2}$$

where λ = cosine of angle between member 1-2 and $0X$, μ = cosine of angle between member 1-2 and $0Y$, u_1 = displacement of node 1 parallel to $0X$, v_1 = displacement of node 2 parallel to $0Y$, and so on. In compact matrix form Eq. (2) becomes

$$\{X\} = [K]\{u\}, \tag{3}$$

in which $[K]$ is the stiffness matrix for the member.

If the single bar of Fig. 1,b is considered as a structural entity, three displacements in Eq. (2) will have to be specified as zero in order to prevent rigid body motion. The corresponding forces then become reactions. Only one active force component can be applied to this member. However, the reactions and the active displacement component can be readily found from Eq. (2).

Now consider the truss consisting of m pin-connected members. Let the individual stiffnesses of these be $K^{(1)}$, $K^{(2)}$, . . . $K^{(m)}$. Then, for the entire truss,

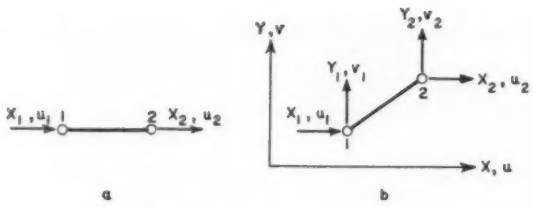

FIG. 1. AXIAL-FORCE MEMBER

$$\{X\} = \begin{bmatrix} K^{(1)} & 0 & 0 & . & . & . \\ 0 & K^{(2)} & 0 & . & . & . \\ 0 & 0 & K^{(3)} & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & K^{(m)} \end{bmatrix} \{u\} \qquad (4)$$

This last equation can be consolidated. The process is not difficult, but need not be explained here. The salient point is that the stiffness of the truss, $[K]_{\text{TRUSS}}$, follows directly from the square matrix of Eq. (4). Consequently, the stiffness matrix for the overall structure can be readily obtained provided the stiffnesses of the individual members are known. Furthermore, a series of matrix calculations then leads to the desired displacements and reactions.

There remains the problem of determining the forces of the internal members. These do, of course, become known when the nodal displacements have been found. In fact, Eq. (2) will enable, for example, X_2 and Y_2 to be calculated for member 1-2 in terms of u_1, v_1, u_2, and v_2. Then the member force S is simply given by

$$S = X_2\lambda + U_2\mu. \qquad (5)$$

All unknowns thereby become known. Note also that it makes no difference what the order of redundancy may be, nor do we have to perform a stress analysis for unit-applied forces or redundants.

Two-Dimensional Continuum

The rectangle of Fig. 2,a is a simple example. Forces and support conditions have not been included.

Figure 2,b shows the same rectangle replaced by a total of 4 triangular subelements joined together at nodes 1,2, . . . 6. No connection between adjacent triangles is assumed except at the nodal points; and the selection of triangles is in no sense unique.

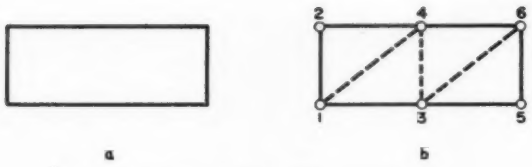

FIG. 2. TWO-DIMENSION CONTINUUM

When this idealization is kept in mind, the continuous rectangle becomes quite similar to the previous truss. If we can determine the stiffness matrix for an arbitrary triangular unit, we can then form $[K]$ for the idealized surface of Fig. 2,b. A complete solution is then possible. This will be an approximation to the true solution for the actual problem. As more triangles are used to represent the given surface, the accuracy of the results will improve. Experience indicates that satisfactory data can be obtained with a reasonable number of subunits.

Many additional statements can, and probably should, be made. For example, others besides triangular shapes can be used for the subunits, although triangles are the simplest to handle and give acceptable results. The density of nodes can also be varied so as to be a maximum in regions of maximum complexity or interest to the analyst. Great flexibility is therefore available when replacing the continuous surface with a finite assemblage of subunits.

Stiffness Matrix for the Triangle

The problem is to find the stiffness matrix $[K]$, which relates the six force components X_1, Y_1, . . . to the six corresponding displacements, u_1, v_1, u_2, . . . This is an interesting problem, for which the solution first appeared in Reference 1. The arbitrary triangular element is illustrated in Fig. 3.

It is evident that the concentrated node forces will cause stress singularities in the triangle. These will, of course, be at variance with the usual state of stress existing in the triangular subelement replacing the continuum. Therefore the nodal forces must be viewed as a useful but fictitious representation for a reasonable state of stress in the element.

To accomplish this we begin by assuming constant strain in the triangle, or

$$\epsilon_x = \frac{\partial u}{\partial x} = a,$$

$$\epsilon_y = \frac{\partial v}{\partial y} = b,$$

and

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = c. \qquad (6)$$

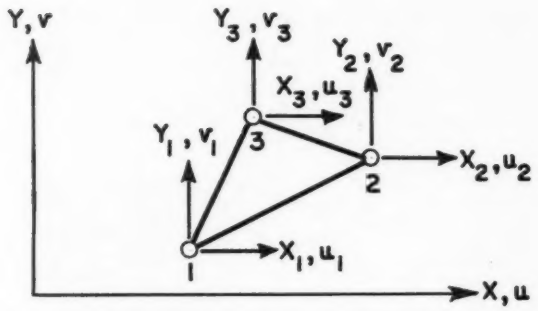

FIG. 3. ARBITRARY TRIANGULAR ELEMENT

Integration gives,

$$u=ax+(c-d)y+f$$

and (7)

$$v=dx+by+e.$$

Constants d, e, and f are due to integration. The six constants agree with the total number of nodal displacements. This is necessary to the analysis that follows.

We note that when $\epsilon_x=\epsilon_y=\gamma_{xy}=0$

$$u=-dy+f,$$

$$v=\ \ dx+e,$$

and

$$2\omega_0=\left|\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right|_{a=b=c=0}=d.$$

Hence d represents rigid body rotation, while e and f represent rigid body translation in the absence of rotation.

It is now possible to write u, v at the nodes. This gives six equations that can be solved for the six constants in terms of nodal displacements and coordinates locating the vertices of the triangle. Furthermore, the strains can now also be expressed in terms of nodal displacements. Let this equation be given by

$$\{\epsilon\}=[\Theta]\{u\}. \tag{8}$$

Hooke's law relates stresses and strains. The equations will be expressed as

$$\{\sigma\}=[\mathcal{C}]\{\epsilon\}. \tag{9}$$

In the above equations only constant elements appear in $[\Theta]$ and $[\mathcal{C}]$.

The strain energy for the triangle can now be written as

$$U=\tfrac{1}{2}\int_{\text{vol}}\{\sigma\}'\{\epsilon\}d(\text{vol.}),$$

where the integration is over the volume of the element. Substituting into this last equation from Eq. (8) and Eq. (9) gives

$$U=\tfrac{1}{2}\int_{\text{vol}}\{u\}'[\Theta]'[\mathcal{C}]'[\Theta]\{u\}d(\text{vol}). \tag{10}$$

The integrand is a constant; hence U is known.

Applying Costigliano's Theorem permits nodal forces per unit displacement to be calculated. This determines the required stiffness matrix as follows:

$$\left|\frac{\partial U}{\partial\{u\}}\right|_{\{u\}=\{1\}}=[K][\Theta]'[\mathcal{C}]'[\Theta]\times\text{Vol triangle}. \tag{11}$$

As given in Reference 1, the result of the calculation just completed is the stiffness matrix for the arbitrary triangle of Fig. 3. As calculated from Eq. 11, this matrix is as follows:

$$[K]=\frac{Et}{2}\phi\begin{bmatrix}
 & u_1 & v_1 & u_2 & v_2 & u_3 & v_3\\
u_1 & \lambda_1x_{23}^2+y_{23}^2 & — & — & — & — & —\\
v_1 & \lambda_2x_{32}y_{23} & x_{23}^2+\lambda_1y_{23}^2 & — & — & — & —\\
u_2 & \lambda_1x_{23}x_{31}+y_{23}y_{31} & \lambda_1x_{13}y_{23}+\nu x_{23}y_{13} & \lambda_1x_{13}^2+y_{13}^2 & — & — & —\\
v_2 & \lambda_1x_{23}y_{13}+\nu x_{13}y_{23} & x_{23}x_{31}+\lambda_1y_{23}y_{31} & \lambda_2x_{13}y_{31} & x_{13}^2+\lambda_1y_{13}^2 & — & —\\
u_3 & \lambda_1x_{12}x_{23}+y_{12}y_{23} & \lambda_1x_{21}y_{23}+\nu x_{32}y_{12} & \lambda_1x_{12}x_{31}+y_{12}y_{31} & \lambda_1x_{12}y_{13}+\nu x_{13}y_{12} & \lambda_1x_{12}^2+y_{12}^2 & —\\
v_3 & \lambda_1x_{32}y_{12}+\nu x_{21}y_{23} & x_{12}x_{23}+\lambda_1y_{12}y_{23} & \lambda_1x_{13}y_{12}+\nu x_{12}y_{13} & x_{12}x_{31}+\lambda_1y_{12}y_{31} & \lambda_2x_{21}y_{12} & x_{12}^2+\lambda_1y_{12}^2
\end{bmatrix}$$

where $\phi = \frac{1}{2(1-\nu^2)A}$

A = area of triangle

t = thickness of triangle

$\lambda_1 = \frac{1-\nu}{2}$, $\lambda_2 = \frac{1+\nu}{2}$,

$x_{ij} = x_i - x_j$, and so on.

Since $[K]$ is symmetric, the elements represented by — can be written at once by inspection.

The matrices required in stress calculations as given by Eq. 12 are

$$[\mathcal{E}] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \cdots$$

and,

$$[\Theta] = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \cdots$$

where, as before,

$$x_{ij} = x_i - x_j \text{ and } y_{ij} = y_i - y_j .$$

Once again we need to keep in mind that the nodal forces associated with the arbitrary triangle are in a sense fictitious. This is true to the extent that, acting through the corresponding nodal displacements, they produce a state of constant strain (and stress) in the triangle.

An important point, which was not mentioned in Reference 1, is that, since u and v are linear in x and y, Eq. (7), common edges of adjacent triangles will remain common after deformation has occurred. Hence continuity is preserved between nodes. However, strains are restricted as given by Eq. (6).

Finally, having found $[K]$ for the assemblage of triangles to determine node displacements, the problem remains to find the stresses. This is readily accomplished by substituting Eq. (8) into Eq. (9), to get

$$\{\sigma\} = [\mathcal{E}][\Theta]\{u\}, \qquad (12)$$

where $[\mathcal{E}]$ and $[\Theta]$ are known.

The stresses thereby determined for any triangular subelement are uniform over the triangle. However, stresses will vary between triangles. Various schemes will occur to the analyst for using these results for plotting or otherwise estimating the continuous stress variation in the actual member.

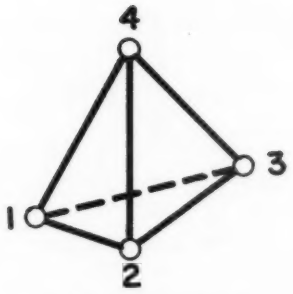

FIG. 4. TETRAHEDRON

Three-Dimensional Continuum

Before proceeding to examples illustrating the use of the stiffness matrix for the triangular element, extension of the previous theory to the solid will be briefly discussed. Although these extensions have not been worked out in detail as yet, they are logical developments beyond the planar case.

The basic element is taken as the tetrahedron (Fig. 4). A total of 12 components of nodal displacement can now exist. If we again choose constant strains, we have

$$\epsilon_x = \frac{\partial u}{\partial x} = a \ , \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = d \ ;$$

$$\epsilon_y = \frac{\partial v}{\partial y} = b \ , \quad \gamma_{xy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = e \ ; \qquad (13)$$

and

$$\epsilon_z = \frac{\partial w}{\partial z} = c \ , \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = f \ .$$

Integrating to obtain displacements leads to

$$u = ax + gy + (f-k)z + l \ ,$$

$$v = (d-g)x + by + (e-h)z + m \ , \qquad (14)$$

and

$$w = kx + hy + cz + n.$$

The above displacement equations contain 12 arbitrary constants. Consequently the chosen strain expressions will enable the stiffness matrix to be

(Continued on Page 19)

A Rational Analytic Theory of Fatigue

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Research Engineers, Boeing Airplane Company

P. C. Paris

M. P. Gomez

W. E. Anderson

A great deal of effort has recently centered around examination of the factors influencing the growth of fatigue cracks. Fatigue has been considered a multiphase problem: e.g., initiation of a crack and its growth are often considered as separate phenomena. In contrast, the objective of this work is to show that the growth of an initial "crack-like" imperfection to a critical size, which causes static failure of a structure, may be described by a single rational theory.

Two loading parameters, the nature of the stress field near the tip of a crack and the variation of this field, are taken to control the rate of crack extension in a given material. This hypothesis is proven by using it to correlate data from three independent investigators. Since it shows a positive correlation of all available data for crack-extension rates from 10^{-7} to 10^{-2} in. per cycle, the hypothesis may be used to formulate a theory of fatigue that permits computing the structural lives of complicated geometries from simple laboratory tests of material properties.

The Stress Distribution Near the Tip of a Crack

The form of the stress distribution in the vicinity of a crack root was given by Sneddon[1] in 1946 and has recently been expanded by Irwin[2,3] and Williams.[4] The unique character of this form, as Irwin showed,[2] is a controlling factor in attempts to analyze crack extension under static loads. We will show that this same character becomes fundamental in crack extension under cyclic loading upon the addition of new concepts to describe the cyclic nature of the loading.

* Mr. Gomez received his M. S. degree in Metallurgical Engineering in 1958 at the University, after which he worked for Boeing. He is now Senior Scientist at the Missile Systems Division of Lockheed Aircraft Corporation.

Restricting this discussion to cracked bodies in which the geometry and loading of the body are symmetric with respect to the plane of the crack results in very little loss of generality. The nature of cracks

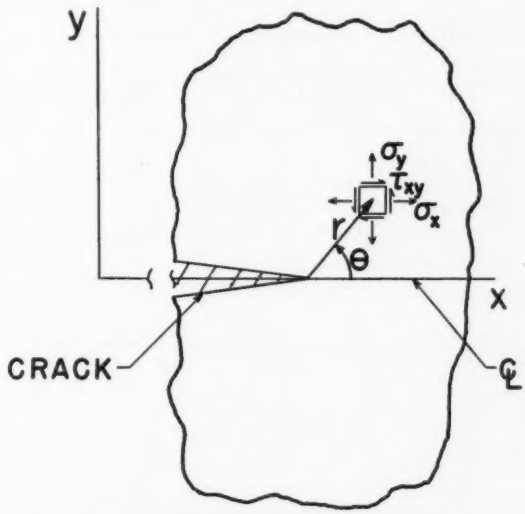

FIG. 1. COORDINATES USED TO DESCRIBE STRESSES NEAR A CRACK TIP (θ; σ_x; τ_y; v)

is to form most often on such planes, i.e., planes perpendicular to maximum-principle tension stresses. Williams[4] and Irwin[5] have given the required forms of stresses for other cases, but these will not be discussed further in this work.

The coordinates of points in a cracked body with

SYMBOLS

a	= the half crack length
b	= plate dimension parallel to a crack
F	= a loading force on a body
$f()$ or $F()$	= a function of
K	= the stress singularity-intensity factor at the tip of a crack
K_{cr}	= the critical value of K for a material associated with a crack extension under static load
L	= the length of a plate
N	= the number of load cycles since initial loading
P	= the loading of a body
r, θ	= polar coordinates from the crack tip
x, y	= rectangular coordinates centered with respect to a crack
α	= the correction factor for K in plates of finite width
β	= the ratio of maximum to minimum load on a body during a load cycle
μ	= Poisson's ratio
σ_g	= gross area stress or nominal stress level
$\sigma_x, \sigma_y, \tau_{xy}$	= components of stress near a crack tip
$\sigma_o(x)$	= the normal stress present at a crack location before the crack appeared
$\Delta a/\Delta N$ or da/dN	= the rate of crack extension

crack-plane symmetric geometry may be described as in Fig. 1. If terms of higher order in r are ignored, the elastic solution for stresses in the vicinity of the crack tip for all such problems is

$$\left.\begin{aligned}
\sigma_y &= \frac{K}{\sqrt{2r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right],\\
\sigma_x &= \frac{K}{\sqrt{2r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right],\\
\tau_{xy} &= \frac{K}{\sqrt{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2},\\
\tau_{xz} &= \tau_{yz} = 0,
\end{aligned}\right\} \quad (1)$$

and

$$\sigma_z = 0 \text{ (plane stress)}$$

or

$$\sigma_z = \mu(\sigma_x + \sigma_y) \text{ (plane strain)}$$

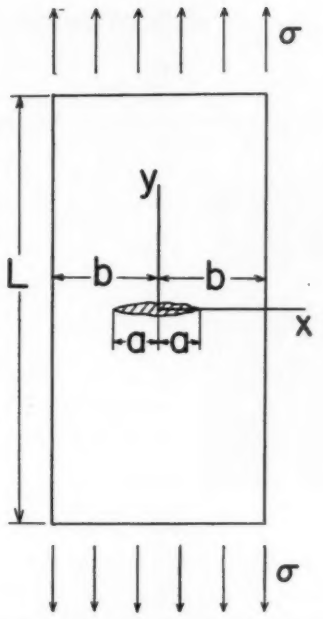

FIG. 2. A TYPICAL FATIGUE-PANEL CONFIGURATION FOR WHICH TEST DATA WERE AVAILABLE

This result implies that the distribution of elastic stress always has the same functional form near the singularity caused by the sharp crack root and that it differs only by a stress singularity-intensity factor, K. This factor is linearly dependent upon the loads on the body and also must contain a geometric factor related to the crack length and other geometric properties of the body. An example is $K = \sigma\sqrt{a}$ for the problem of a crack of length $2a$ in an infinite sheet under a uniform tensile-stress field, σ, perpendicular to the crack.

Now the Griffith-Irwin theory of static strength of bodies containing cracks may be resolved from this discussion in the following fashion: Identical intensity factors of elastic stress will result in identical yield zones near the tips of cracks in the same material if the yield zones are small compared to the region of applicability of the stresses given by Eq. (1). As discussed in previous works,[6] the size of the yield zone may be shown to be small if the nominal stresses in the body are well below the yield point. Therefore, regardless of the appearance of a small yield zone, there will be some critical value of the stress singularity intensity, K_{cr}, near the tip of a crack that will cause static crack extension in a given material. The above hypothesis is the equivalent of the Griffith-Irwin theory, which was originally based on energy considerations.

The Significance of Stress-Intensity Factors in Fatigue

The stress-intensity factor may be considered to be a measure of the effect of the loading and the geometry of a body on the stress intensity near the root of a crack. Therefore, as the loads on a body vary and as the geometry changes by crack extension, the instantaneous values of K reflect the effects of these changes at the crack root.

Let β be the ratio of maximum to minimum load on a cracked body during a cycle of loading. Then, since K is directly proportional to the magnitude of the load, β is also the ratio of K_{max} to K_{min}, regardless of the geometry of the body; that is,

$$\beta = \frac{P_{max}}{P_{min}} = \frac{K_{max}}{K_{min}}. \qquad (2)$$

Therefore the stresses near the root of a crack are completely described by K_{max} and β in a given material, since these two parameters give both the intensity and variation of the effects of loading and geometry.

A theory of fatigue crack extension may now be hypothesized as follows: Since, as has been shown, during a cycle of loading the stresses and strains near the tip of a crack are completely specified by K_{max} and β, we can reasonably assume that any phenomena occurring in this region are controlled by these parameters. The amount of crack extension per cycle of loading is just such a phenomenon, or, in functional form,

$$\frac{\Delta a}{\Delta N} = f(K_{max}, \beta). \qquad (3)$$

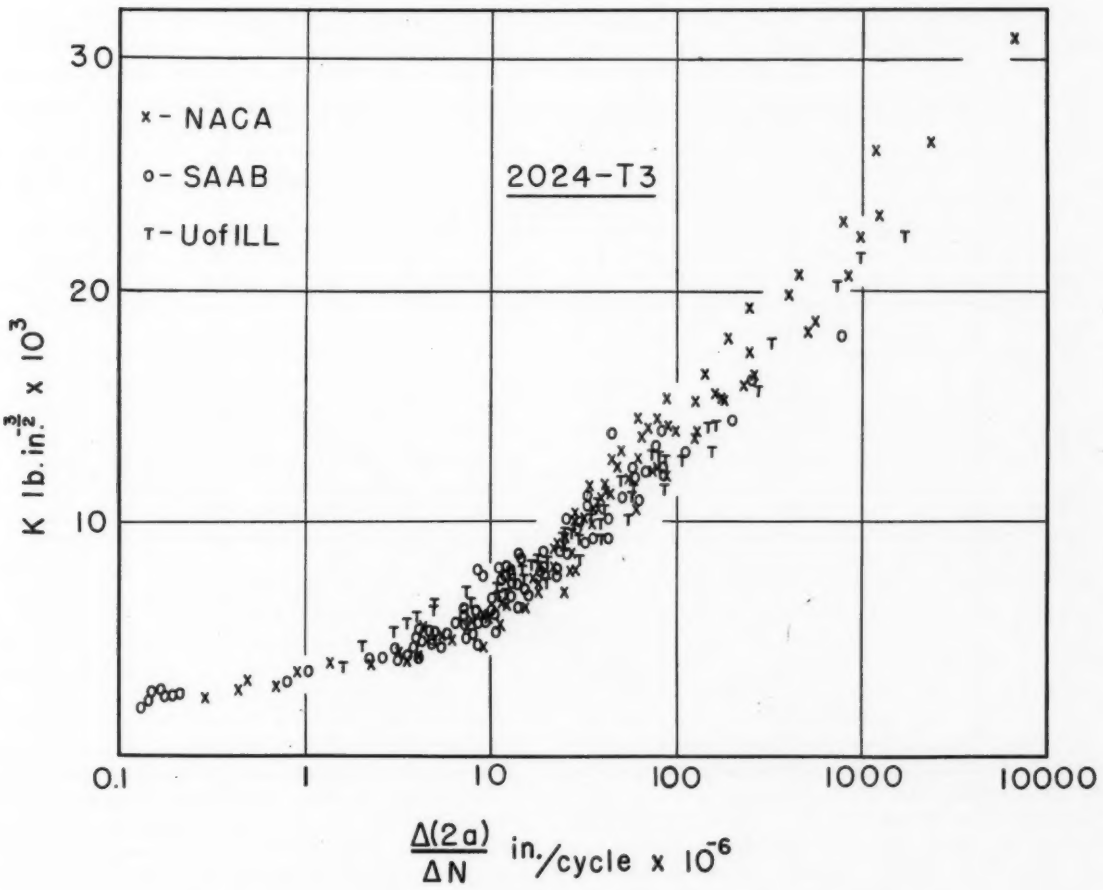

FIG. 3. CRACK EXTENSION-RATE DATA ON 2024-T3 ALUMINUM ALLOY CORRELATED FROM TESTS BY THREE INDEPENDENT INVESTIGATORS

Experimental Evidence

It is pertinent to examine the above hypothesis in the light of available experimental data. Many investigators[7,8,9,10] have measured crack-extension rates due to cyclic loading in aluminum alloys by using the configuration shown in Fig. 2. The majority of these tests have been performed with minimum loads near zero, or $\beta \equiv \infty$; therefore from examination of Eq. (3), these results for a given material should form a single curve on a plot of K_{max} *vs* $\Delta a/\Delta N$. For such a configuration K_{max} may be computed from[3]

$$K = \alpha \sigma_g \sqrt{a},$$

where

$$\alpha = \frac{\sqrt{4 + 2\,(a/b)^4}}{2 - (a/b)^2 - (a/b)^4} \qquad (4)$$

if

$$a < b.$$

The results of this attempt at correlation of crack extension-rate data are shown on Figs. 3 and 4. It is worthy of special note that the data on these curves are from three independent investigators, using many specimen sizes, i.e., widths from 1.8 to 12 in., thicknesses from 0.032 to 0.102 in., and lengths from 5 to 35 in. The testing frequencies varied from 50 to 2000 cpm, and the maximum stresses on the gross area varied from 6 to 30 ksi. On each graph, the materials are both clad metals and bare metals. Therefore the correlation shown is surely more than coincidental.

On the presumption that such curves may be obtained for various values of β for a given material from laboratory tests, this discussion proceeds to formulate the necessary elements of an analytic theory of fatigue, these results being applied in the following sections.

An Analytic Theory of Fatigue

Knowledge of the material-property curves in the form of Figs. 3 and 4, with the addition of curves for other β values, implies the functional form of Eq. (3) as given. Further, given the loading and geometry

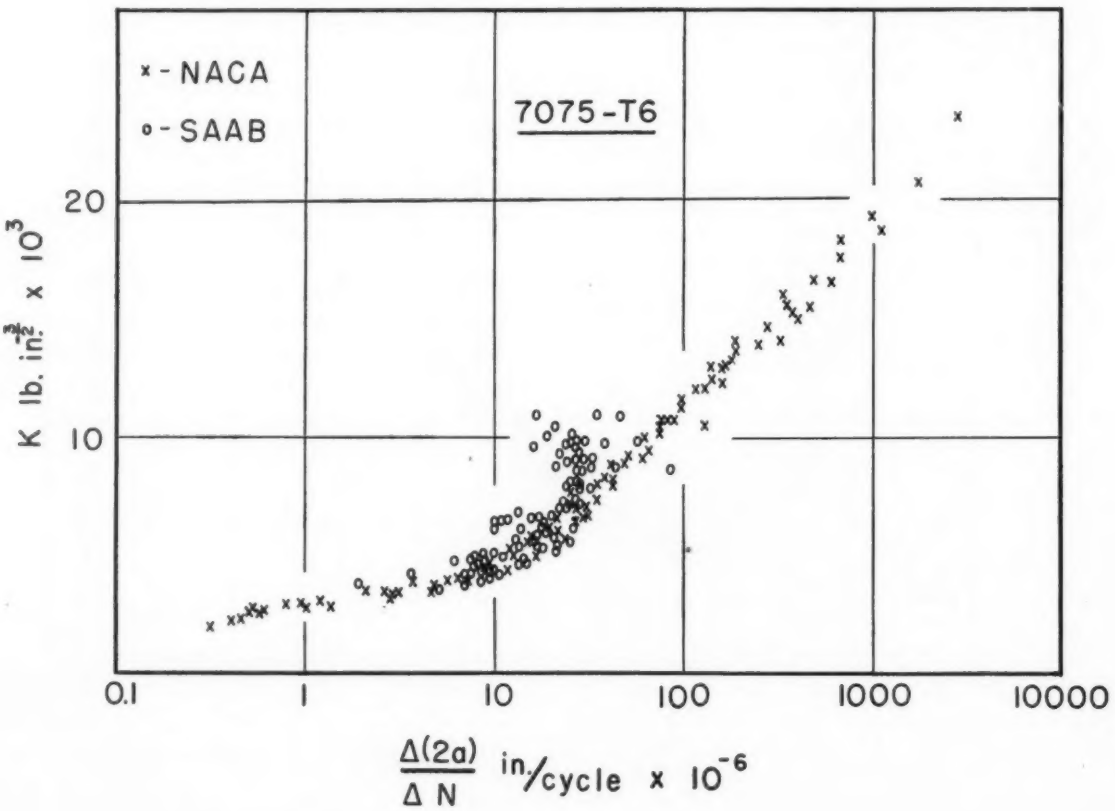

FIG. 4. CRACK EXTENSION-RATE DATA CORRELATION FOR 7075-T6 ALUMINUM ALLOY

of a structure, β is known from the load ratio during any cycle, N, or

$$\beta = \beta(N). \tag{5}$$

Moreover, K may be computed for any crack length, a, and the maximum load as given during the Nth cycle, or

$$K_{max} = K_{max}(N,a). \tag{6}$$

Therefore, for a problem with the above specification, Eq. (5) and Eq. (6) may be substituted into Eq. (3) to give

$$da/dN = F(N,a), \tag{7}$$

where the functional form of F is known, point by point, from the data given, and effects of loading history are neglected.

The solution to Eq. (7) may be found provided an initial crack size may be specified, or some equivalent condition may be stated. In practice, maximum imperfection sizes may be stipulated on the basis of material quality, production methods, and inspection technique, for in practice we know that fatigue cracks grow from just such imperfections. Then Eq. (7) may be integrated, at least by numerical procedures, to generate a complete crack history for the structure. The most difficult phase of this analysis is the computation of K for a given load and crack length, as will be commented upon later.

Relationships to Classical Fatigue Theory

Suppose the material-property curves of the form of Figs. 3 and 4 are known for a specimen that has been subjected to an ordinary fatigue test and has developed an easily measurable crack in a given number of cycles. By using these curves and the measured crack length, the crack-extension rates may be integrated backward to determine an effective initial imperfection size.

Using this computed initial imperfection size makes it possible to calculate the number of load cycles required for failure of the specimen at any stress level. Therefore the complete S-N curve for a material may be computed by this process; moreover, the whole S-N curve can be obtained from a single specimen, since curves of the material properties for several β values may be generated during the same test.

An accumulative damage theory is automatically present in the preceding analysis, which replaces Miner's empirical hypothesis of damage. The Soderberg or Goodman diagrams being likewise empirical, this paper presents a rational analytic theory. As is evident, the form of the analysis lends itself well to statistical analyses.

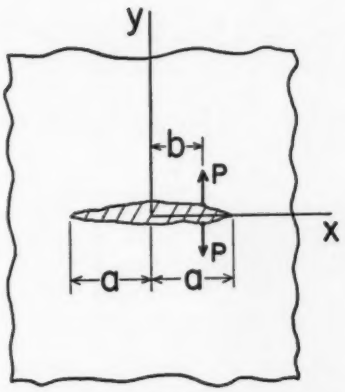

FIG. 5. A CRACK IN AN INFINITE SHEET WITH CONCENTRATED FORCES APPLIED TO CRACK SURFACE

Computation of K, the Stress-Intensity Factor

By use of data on materials, in the form of Fig. 3, the inherently nonlinear problem of structural life has been resolved to require only the computation of instantaneous values of K as the crack propagates through the structure. Since K is the elastic-stress intensity factor for stresses near a crack tip, the problem has been essentially linearized from the point of view of mechanics, a method that permits the use of well-known techniques in attacking the problem.

Irwin[3] has given the solution to the problem shown in Fig. 5, which may be applied by superposition technique (as a Green's Function) to solve all problems of sheet-skins containing internal cracks. This method, which is described in detail in Reference 6, will be outlined here.

Consider the general and arbitrary x-axis symmetric plane-stress problem containing a crack, illustrated by Fig. 6(a). This may be considered to be a free body removed from a gross structure, and may be chosen large enough in size that the crack itself has very little influence on the magnitude and distribution of the boundary forces and stresses shown. Thus we may proceed to solve the plane-stress problem for this sheet, but we find considerable difficulty in problems of doubly connected regions. As an alternative, this problem may be considered to be the sum of two problems, i.e., those in Fig. 6 (b) and (c). The first, (b), is the solution to the same problem with no crack present; the second, (c), is the solution with the same geometry as the original problem, (a), but with loads on the crack surfaces only, of equal and opposite intensity to the stresses that occur at the crack location in (b). This approach ensures that the sum of the boundary forces in (b) and (c) are identical to those in (a), and thus their sum is the solution to (a).

Now we desire only to determine the stress inten-

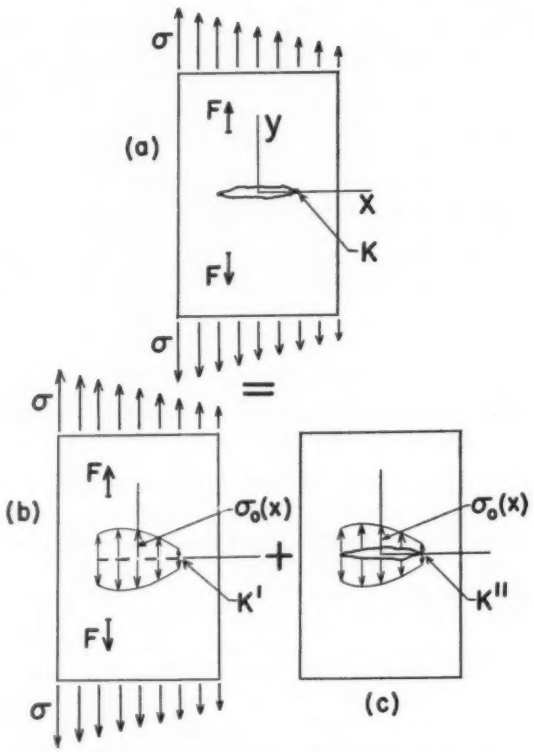

FIG. 6. THE REDUCTION OF A PROBLEM, (a), INTO TWO SIMPLER PROBLEMS, (b) and (c), FOR COMPUTATIONS OF STRESS SINGULARITY - INTENSITY FACTORS

sity factor, K, for the problem, which is the sum of the stress intensity factors, K' and K'', for problems (b) and (c). Since K is the intensity of the singularity of stresses at the crack tip—note $1/\sqrt{r}$ in Eq. (1)—and this singularity is not present, in (b), then

$$K' = 0. \tag{8}$$

The task remaining is to determine K'' for problem (c), since, observing Eq. (2),

$$K = K''. \tag{9}$$

Thus the solution to all problems of the type of (a) is reduced to finding the stress, σ_o (x), on the crack axis with no crack present, as in (b), and using this stress as the loading in (c).

To find the value of the stress singularity, K'', for (c), Irwin's solution[3] for the stress singularity in Fig. 5 gives

$$K_P = \frac{P(a+b)^{\frac{1}{2}}}{\pi(a)^{\frac{1}{2}}(a-b)^{\frac{1}{2}}}. \tag{10}$$

Taking (10) as a Green's Function, the stress singularity, K'', is given by

$$K = K'' = \frac{1}{\pi\sqrt{a}} \int_{-a}^{a} \frac{\sigma_o(x)\,(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}\,dx, \tag{11}$$

which may be further simplified by *y-axis symmetry*,

$$K = \frac{2\sqrt{a}}{\pi} \int_{o}^{a} \frac{\sigma_o(x)}{(a^2-x^2)^{\frac{1}{2}}}\,dx. \tag{12}$$

Hence all the general problems of the form of Fig. 6(a) may be attacked by solving the problem for the stresses, σ_o (x), along the crack axis with the crack absent, and integrating the result by Eq. (11) or Eq. 12.

Equally powerful techniques may be presented for other general classes of problems. Therefore we conclude that, in general, K may be computed easily in most of the problems of interest for engineering purposes.

Conclusion

On the basis of the experimental data given, it is evident that rates of crack growth—for example, those in 2024-T3 and 7075-T6 skins of aircraft structure—may be computed by the theory presented over a wide range of nominal stress levels and crack sizes. The ramifications of such broad correlation imply an analytic theory of fatigue based on a concept of growth from initial imperfections through which structural life may be predicted.

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High-Temperature Metallography

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Metallography is the branch of metallurgy that relates the microstructure of metals and alloys to their physical and mechanical properties. Conventional metallographic techniques have provided the metallurgist with much valuable information concerning solid-state reactions that accompany physical and mechanical property changes in materials subjected to unusual environments. However, with conventional techniques it has been necessary to base predictions of high-temperature behavior on metallographic observations made at room temperature. The development of practical methods for direct examination of the microstructure of metals at elevated temperatures has provided a new approach to the study of the high-temperature constitution of alloys and of microstructural processes.

The structure observed under the metallurgical microscope must be truly representative of the parent material if the conclusions drawn from the study are to be valid. Traditionally, the procedure involves preparation of an optically smooth surface followed by selective etching to reveal structural details such as crystal boundaries and phase distributions. Since the properties of many alloys are dependent on structural changes that occur at elevated temperatures, studying the structures that exist in those temperature ranges would be desirable. In such studies, common practice is to heat a specimen to the temperature range desired, hold it until equilibrium is essentially achieved, and then rapidly quench the specimen to room temperature. The assumption is made that the rapid rate of cooling causes the high-temperature structure to be retained at room temperature, thus allowing the examination to be accomplished.

The shortcomings of such a procedure are evident. The microstructure observed at room temperature after quenching may not represent the high-temperature structure if a phase transformation occurs during cooling. Martensitic transformations or rapid precipitation reactions may completely mask the elevated temperature constitution of an alloy. In addition, the preparation of a metallographically smooth surface usually involves mechanical polishing, which can alter the structure of some metastably retained phases by deforming the surface layers.

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Owing to the problems outlined above, it is understandable that metallographers have long wished for a facility that would permit direct observation of the microstructure at elevated temperatures. In addition to eliminating the sources of error just mentioned, such a facility would permit continuous observation of the same field of view over a wide range of temperatures. Thus the mechanisms underlying structural changes could be studied by direct observation of the microstructural changes accompanying phase transformations. Such a facility, called a "hot-stage" metallograph, is now in operation in the Division of Metallurgical Engineering of the School of Mineral Engineering. The equipment is currently being used in a study of phase transformations in the copper-silicon alloy system. This paper will describe the facility and illustrate the use presently being made of it.

Principles of High-Temperature Metallography

Before proceeding with a description of the high-temperature metallograph, we should review some basic metallographic principles. It will be subsequently shown that these considerations are influential in the design of high-temperature metallographic equipment.

When ordinary "bright field" illumination is used, the light beam strikes the specimen essentially normal to the surface, and observation is made by means of light reflected into the objective. When the surface has elevations and depressions resulting from selective attack by etchants, these structural details can be resolved as a result of differences of intensity of the light reflected from the surface irregularities. For example, the presence of a surface elevation may indicate the presence of a phase not greatly attacked by etchants, whereas a valley may be due to a phase with a higher rate of solution or to a crystal boundary. Thus, for bright-field examination, the surface

contour must be altered so that differences in the intensity of light reflected from the structural details of the surface will enable the observer to resolve these details and to see the microstructure.

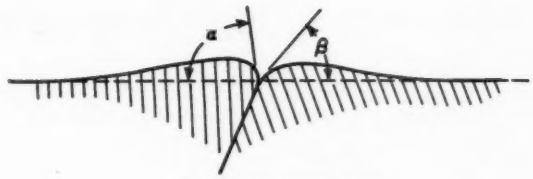
If the light incident on a metallic surface is plane polarized and if the material under study is isotropic (for example, a phase of cubic structure), then the reflected light will also be plane polarized. By proper placement of a polarizing filter in the microscope objective, extinction of reflection from such surfaces can be obtained. However, if the structure also contains an anisotropic phase (for example, a phase of hexagonal structure) then complete extinction will not occur because light reflected from such phases is no longer plane polarized. A resolution of separate phases may thus be achieved.

The principles by which a structure is revealed under bright field or under polarized light are applicable regardless of the temperature at which the observation is made. If bright field illumination is used, the surface must have a contour that reflects the underlying structure or, if polarized light is used, the desired distinction must be between isotropic and anisotropic phases.

There are essentially three ways in which heating can produce contour on a metallographically smooth metal surface: (1) thermal etching, (2) surface tilting, and (3) surface upheaval.[1]

Thermal etching occurs when a metal or alloy is heated to a sufficiently high temperature in a vacuum or in a protective atmosphere. It occurs where a grain boundary or an interface between phases intersects the surface of the specimen. It can also occur where a plane of low surface energy exists near the surface. The mechanism of grain-boundary etching is believed to involve the establishment of equilibrium surface tension between the intersecting boundaries. Thus the angles *alpha* and *beta* of Fig. 1a are established by means of surface diffusion or by evaporation from the boundary and deposition adjacent to it. In a somewhat similar fashion, net evaporation may cause low-surface-energy planes to develop by transport of metal through the vapor phase, provided that means exist for removal of the vapor from the system.[2] This may be accomplished by a dynamic vacuum system or by condensation on cooler parts of a furnace.

Surface tilting and surface upheaval are similar in that both reflect a change in volume, concomitant with the phase change. The distinction is made on the basis of the shape of the surface after the transformation. Surface tilting, illustrated in Fig. 1b, occurs when the transformation is of the "martensitic" type. In this transformation, rows of atoms suddenly shear across each other, producing a rotation in addition to a volume increase. Surface upheaval, illustrated in Fig. 1c, reflects a transformation known as a "nucleation and growth" type. The surface contour produced by this type of transformation is not well defined because of variations in types of transformation products, in their shapes and in the locations at which nucleation occurs.



a. THERMAL ETCHING

b. SURFACE TILTING

c. SURFACE UPHEAVAL

FIG. 1. THE THREE WAYS IN WHICH HEATING PRODUCES CONTOURS ON SMOOTH METAL SURFACES

The microstructure may thus be revealed without the use of chemical etchants, provided that either thermal etching takes place or that a volume change occurs. While a specimen cannot be etched under vacuum at elevated temperature by using the ordinary chemical etchants, sometimes a gas at a low pressure can be introduced to selectively attack the surface of the specimen and reveal its structure. For example, a small amount of oxygen may be introduced to form oxide layers of varying thickness on steel alloys, thus permitting phase resolution.

Hot-Stage Metallographic Equipment

The type of equipment required for high-temperature metallography is shown in Fig. 2. The basic components of the facility include a metallograph, a hot-stage furnace, a vacuum system, an inert gas system, and a control panel.[3]

The metallograph, A, is an inverted microscope of the type conventionally used in metallographic studies. An eyepiece is provided for visual examination and provisions are made for photomicrography

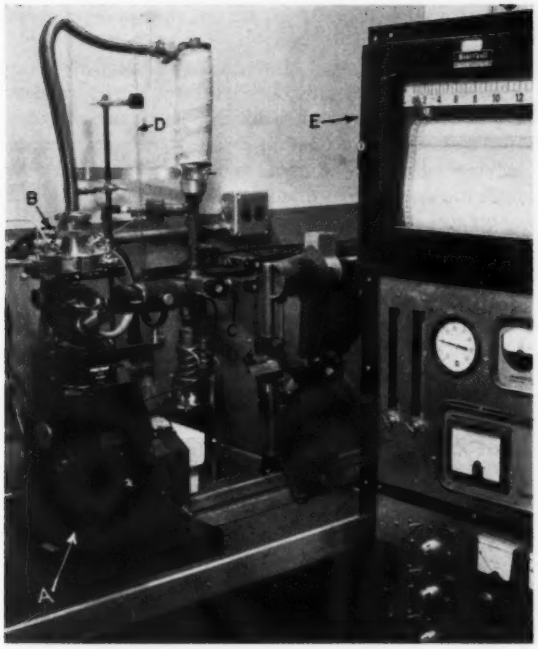


FIG. 2. HOT-STAGE METALLOGRAPHIC EQUIPMENT

with 3¼ x 4¼ in. film. The metallograph is easily adapted for 35-mm still photography or for 16-mm motion-picture photography. The specimen may be illuminated by a tungsten light source attached to the metallograph or by a carbon-arc illumination source when higher intensity is required. The arc illuminator is situated on a track, enabling it to be moved out of the way when the tungsten source is in use.

The hot-stage furnace, B, is bolted to the top of the metallograph stage. Its design permits the specimen to be observed through a quartz window in the bottom of the stage. We shall subsequently return to a more detailed consideration of this unit.

The vacuum unit, C, consists of a conventional diffusional pump and a mechanical forepump. The pressure inside the stage must be reduced to a level where the rate of oxidation of the specimen is negligible over the period of time required for the study. Ultimate pressures between 10^{-4} and 10^{-5} mm of mercury are adequate for most studies. This system is connected to the furnace through a cold trap and a flexible metal hose.

An inert gas such as argon or helium may be used to accelerate the rate of cooling of a specimen that has been heated under vacuum, or an inert gas may be used to protect the specimen during the entire run. When inert gas is used, the pressure is maintained at approximately one atmosphere by use of a manometer-bubbler, D. The inert gas is purified by passing it over heated calcium chips before it enters the furnace.

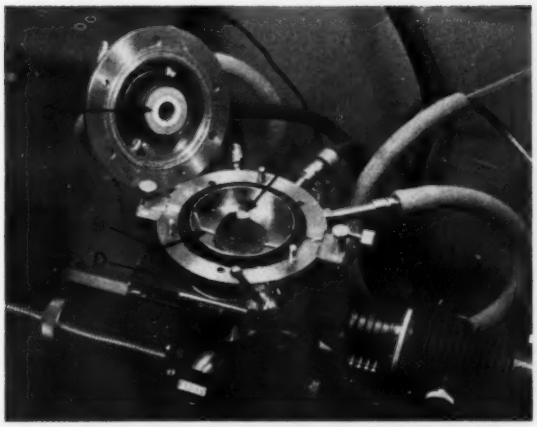


FIG. 3. FURNACE FOR HOT-STAGE METALLOGRAPHY

The control unit, E, was designed and built in the Division of Metallurgical Engineering. It consists of three major sections. The top compartment contains a strip-chart recorder on which the temperature of the surface of the specimen is both indicated and recorded. The center compartment contains the flowmeters for controlling the flow of coolant to the furnace and to the diffusion pump. A thermocouple vacuum gauge and a discharge vacuum gauge are also located in this compartment. The first of these is used for rough vacuum measurement; the second is used to measure the lower pressures. The bottom compartment contains the power supply, switches, and meters.

A closer view of the hot-stage furnace is provided in Fig. 3. Specimen A, a cylinder 10 mm in diameter by 25 mm long, is placed on a quartz window with the surface to be studied positioned downward. Precise temperature measurement is provided by a platinum/platinum-13% rhodium thermocouple, B, resting in a slot in the specimen surface. The specimen is heated by a tungsten resistance element, C, which is lowered over the specimen when the upper and lower portions of the furnace are joined. When operated under vacuum, the element is capable of heating the specimen to 1500° C. Thermal conductivity somewhat reduces the maximum obtainable temperature when an inert gas is used in place of a vacuum.

When a metal or an alloy is heated to high temperature in a vacuum, a temperature-pressure relationship may be reached wherein volatilization of the specimen becomes appreciable. Excessive condensation of volatilization products on the quartz window will obscure the sample. Consequently, a shutter, D,

has been incorporated into the stage. Closing the shutter slips an extra piece of quartz between the specimen and the microscope objective; opening the shutter removes the extra quartz and any condensate from the optical path, making it possible to observe the specimen.

Examples of High-Temperature Photomicrography

An example of photomicrography at elevated temperatures is shown in Fig. 4. This sequence, photographed at 200 diameters, shows the changes that occur in the structure of an as-cast copper-5.3% silicon alloy when it is heated to a temperature where incipient melting has occurred. An etchant has been used to permit observation of the room-temperature structure.

The appearance of the microstructure at room temperature is shown in Fig. 4a. Three distinct phases may be observed: (1) The matrix, alpha, is a solid solution of silicon in copper and has a face-centered cubic crystal structure. (2) The dark constituent has been identified as gamma, which, according to the phase diagram, may have a composition ranging from 8.3 to 9% silicon. Previous investigators [4,5] have found that the gamma phase has a complex crystallographic structure of the beta-manganese type. The amount of gamma in the microstructure is greater than would be expected for a 5.3% silicon

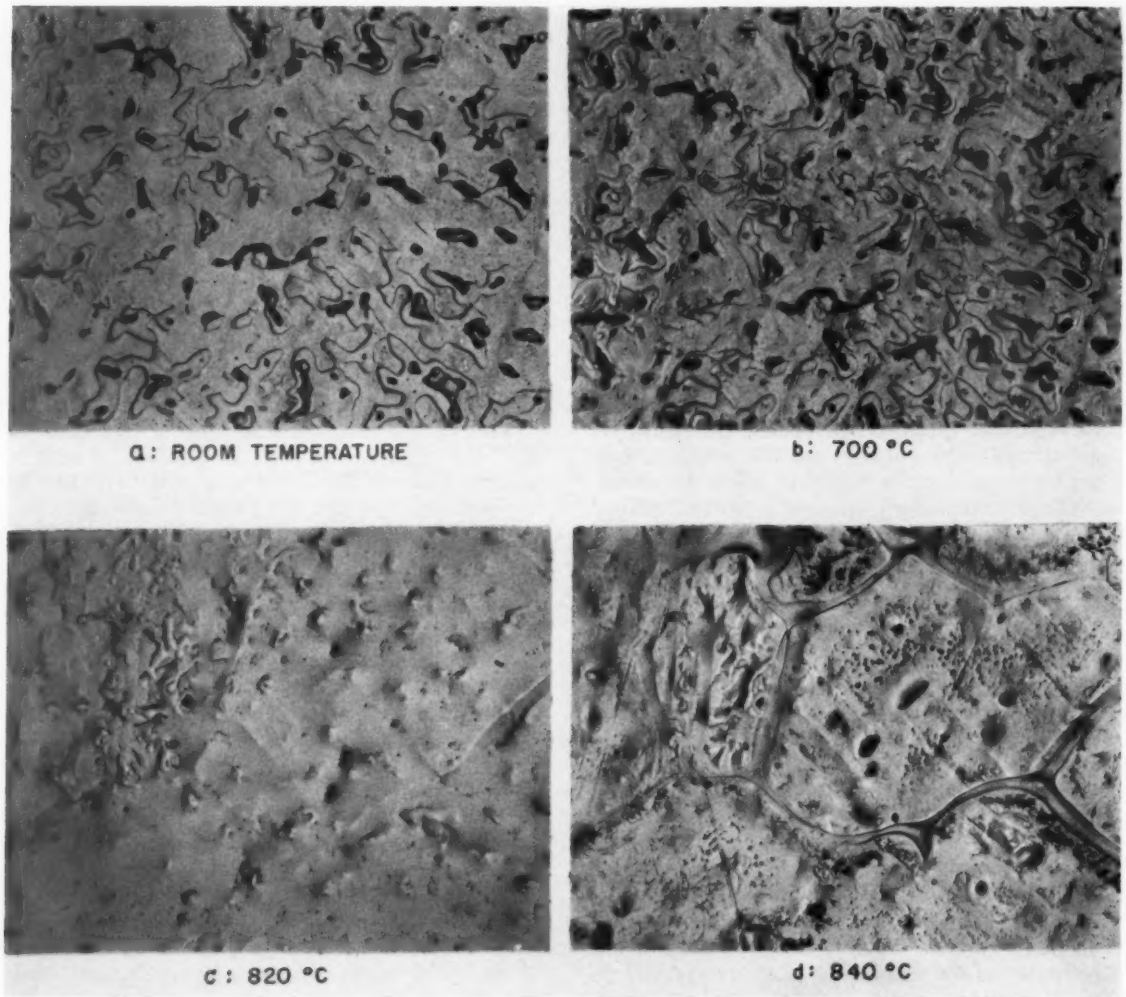


FIG. 4. MICROSTRUCTURAL CHANGES DURING HEATING OF A COPPER-5.3% SILICON ALLOY
Starting Condition As-Cast and Etched (200X)

alloy on the basis of the equilibrium diagram. Non-equilibrium conditions during freezing have probably caused silicon segregation, accounting for the cored appearance of Fig. 4a. (3) The pools surrounding the gamma consist of kappa phase, which has been identified positively by polarized-light examination and by hardness measurements. This phase, which has a hexagonal close-packed structure, occurs in the copper-silicon phase diagram between 5.2 and 6.9% silicon, depending on the temperature. The relatively large quantity of kappa also reflects segregation in the cast alloy.

At 700° C (Fig. 4b), some fine precipitate particles that are not present at room temperature can be seen in the alpha solid solution. The gamma and kappa phases are still in evidence. The precipitate particles exhibited optical anisotropy and are probably kappa phase. At 820° C, (Fig. 4c), a distinct change has occurred in the appearance of the microstructure, reflecting the start of "incipient fusion," i.e., local melting of regions enriched in silicon content due to the coring effects mentioned previously. The grain-boundary fusion is clearly revealed in Fig. 4d.

Conclusions

Research facilities in the Division of Metallurgical Engineering have been substantially increased through the acquisition of the equipment described, enabling the microstructure of metals and alloys to be examined at elevated temperatures. This equipment is currently being used in a study of elevated temperature transformations in the copper-silicon system. Subsequent studies will involve other alloy systems. The technique will be extremely valuable in advancing basic understanding of the mechanisms of phase changes in a variety of alloys.

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DIRECT STIFFNESS METHOD

(Continued from Page 8)

calculated along lines similar to these employed for the triangle. Moreover, continuity between common faces of adjacent tetrahedra will be preserved after deformation due to the linear form of Eq. (14).

Modern computing equipment has already reached a state of development that makes it reasonable to employ an assemblage of tetrahedra for analyzing three-dimensional problems.

Examples

To date, applications of the stiffness method have been principally directed toward large-scale aircraft structural problems. As a result, problems of the continuum, as such, have received only minor attention. Nevertheless, a useful paper on the subject has already been published. (See Reference 2.)

Figure 5 illustrates three problems investigated in Reference 2. Cases a and b are rather obvious; c represents the cross section of a dam. For each of these problems, deformations as well as stresses were determined by using the stiffness method.

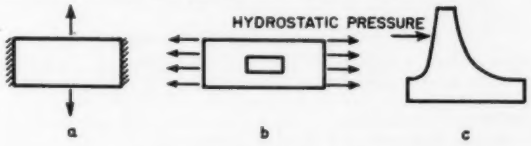

Fig. 5. Two-Dimensional Continua Problems (See Ref. 2)

Results for problem a are compared against a known series solution. Agreement is shown to be quite satisfactory, even when relatively few sub-elements were used to represent the continuum. Case b illustrates a problem difficult to treat by the methods of elasticity. Use of the stiffness method would make entirely possible the inclusion of additional complicating features such as taper in width and thickness. Problem c is analyzed for both dead loads and hydrostatic pressure. The results are shown to agree satisfactorily with previously calculated and published data based on the relaxation procedure. (See Reference 2.)

Additional calculations, presently unpublished, confirm the conclusions reached in Reference 2, namely, that the stiffness method provides a useful and versatile tool for investigating planar continua problems.

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